Mechanism Design for Land Acquisition

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The Problem

- *Land acquisition* : a single buyer purchases a set of contiguous plots from multiple landowners.
- *Role of private information* : If prices acceptable to the agents are publicly known, an efficient outcome is easily achieved: trade takes place whenever the buyer can afford the cheapest set of contiguous plots. When such information is not public, life is not easy.

Main Result

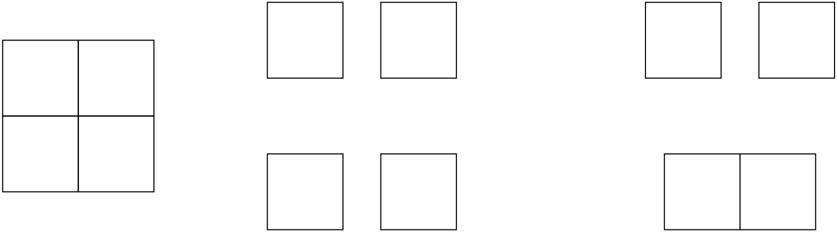
- There exists a robust set of priors for which BIC mechanisms attain the first-best when there are at least two distinct sets of connected plots.
- We identify a sufficient condition and also provide a weaker necessary condition for existence.
- If there is only one set of connected plots, the standard Myerson-Satterthwaite impossibility holds.
- We show that higher the number of critical sellers – sellers who lie in every connected set of plots – the harder it is to satisfy the conditions for possibility.

Background

- Classic impossibility result by Myerson and Satterthwaite (1983): there does not exist a mechanism for bilateral trade that satisfies efficiency, Bayesian incentive compatibility, interim individual rationality and budget balance simultaneously.
- Possibility Results in other contexts:
  - partnership dissolution (Cramton et al, 1987);
  - one seller, two buyer, one indivisible item (Makowski and Mezzetti, 1993);
  - m buyers demanding an item each from n sellers with an item each (Williams, 1999)
- General conditions for possibility: Makowski and Mezzetti (1994), Krishna and Perry (1998), Williams (1999), Schweizer (2006).

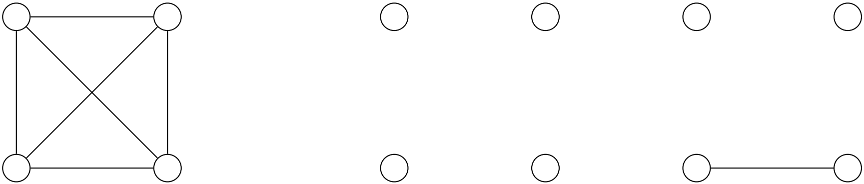
Contiguity

- A pair of plots is contiguous if they share a physical boundary. For example, consider the figures below that show the physical location of four rectangular plots.



Contiguity

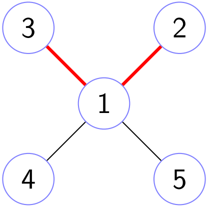
- Each of these situation can be represented by a graph. The nodes in the graph represent sellers. Two nodes are connected by an edge if the corresponding plots are contiguous.



The Model: Graph Structure

- n sellers, indexed by i, each holding one unit of an indivisible good (plot).
- The n indivisible items are located on a graph $\Gamma = (N, E)$ where N denotes the set of nodes (plots) and E denotes the set of edges.
- A pair of nodes is connected by a direct edge if they are physically adjacent to each other.
- A sequence of connected nodes is called a path.
- A path is feasible if it contains at least a fixed number k of nodes where $k \leq n$.

A Feasible Path in a Star Graph when $k = 3$



Critical Sellers

- In this figure, all feasible paths must contain node 1.
- So there may be nodes that are in every feasible path.
- We shall call such nodes in our problem *critical nodes*.

The Model: Valuations

- Valuation of each seller i is $v_i \in [\underline{v}, \bar{v}]$.
- v_i's are independently and identically distributed random variables with distribution function $F(\cdot)$ and density function $f(\cdot)$.
- The realization of v_i is observed only by i.
- One buyer with valuation $v_0 \in [\underline{v_0}, \bar{v_0}]$ if she acquires a feasible path.
- v_0 has distribution function $G(v_0)$ and density function $g(\cdot)$.
- All valuations are non-negative and independently distributed.

The Model: Non-triviality Assumption

- In order to make the problem non-trivial , we make the following assumption:

  ASSUMPTION **NT** : $k\underline{v} < \bar{v}_0$ and $k\bar{v} > \underline{v}_0$

- If the first part does not hold, then the buyer's valuation for any feasible path will always be less than the sum of valuations of the sellers constituting it — trade will never be efficient.
- If the second part is violated, then the buyer's valuation will always exceed this sum of valuations — trade is efficient for any feasible path.

The Model: Notation

- A valuation profile is an $n+1$-vector $v \equiv (v_0, v_1, \ldots, v_n) \in [\underline{v_0}, \bar{v_0}] \times [\underline{v}, \bar{v}]^n$.
- The j-th component of v is denoted by v_j and the n-vector v_{-j} denotes the profile where the j-th component is dropped from v.
- The distribution of the random vector v is called a *prior*, denoted μ.
- A *land acquisition problem* is a tuple $\langle \Gamma, k, \mu \rangle$.

Allocations

- A *deterministic* allocation is an $n+1$-vector x described as follows: for components $i = 1, \ldots, n$, x_i is -1 if seller i sells and 0 otherwise; $x_0 = 1$ if $\sum_{i=1}^{n} |x_i| \geq k$ and 0 otherwise.
- Let $\mathbb{X}$ be the set of all deterministic allocations. Some examples are given below.

Example

Suppose $n = 1$ and $k = 1$. Then, $\mathbb{X} = \{(0,0),(1,-1)\}$.

Example

If $n = 2$ and $k = 2$,
$\mathbb{X} = \{(0,-1,0),(0,0,-1),(1,-1,-1),(0,0,0)\}$.

Direct Mechanisms

Definition (Allocation Rule)

An allocation rule $P : [\underline{v_0}, \bar{v_0}] \times [\underline{v}, \bar{v}]^n \to \mathbb{X}$ maps a profile of reported values to a deterministic allocation.

Definition (Transfer Rule)

A transfer rule t is a map $t : [\underline{v_0}, \bar{v_0}] \times [\underline{v}, \bar{v}]^n \to \mathbb{R}^{n+1}$.

Quasi-linear Utilities

Definition (Payoffs)

Fix a mechanism (P, t). The (ex post) utility of agent j with valuation v_j reporting $\hat{v}_j$ in mechanism (P, t) is

$$U_j^{(P,t)}(\hat{v}_j, v_{-j}|v_j) = v_j P_j(\hat{v}_j, v_{-j}) - t_j(\hat{v}_j, v_{-j}).$$

- Henceforth, we shall fix the mechanism (P, t) and drop the superscript in the notation.

Requirements on Mechanisms

Definition (Bayesian Incentive Compatibility)

A mechanism is Bayesian Incentive Compatible (BIC) if for all j,

$$E_{-j}U_j(v_j, v_{-j}|v_j) \geq E_{-j}U_j(\hat{v}_j, v_{-j}|v_j) \text{ for all } v_j \text{ and } \hat{v}_j,$$

where $E_{-j}(\cdot)$ denotes expectation taken over v_{-j}.

Definition (Interim Individual Rationality)

A mechanism is interim individually rational (IIR) if for all j,

$$E_{-j}U_j(v_j, v_{-j}|v_j) \geq 0 \quad \text{for all } v_j.$$

Requirements on Mechanisms...

Definition (Efficiency)

An allocation rule P is ex post efficient if for all v,

$$\sum_j v_j P_j(v) \geq \sum_j v_j P'_j(v) \text{ for any allocation rule } P'.$$

Definition (Budget Balance)

A mechanism (P, t) satisfies budget balance if, for all v,

$$\sum_{j=0}^{n} t_j(v) = 0.$$

Efficiency in this Model

- Let the feasible paths in G be denoted by $\mathcal{P}_1, \ldots, \mathcal{P}_q$ with $q \geq 1$.
- The sum of valuations in path $\mathcal{P}_i$ will be denoted by $S_i(v)$, $i = 1, \ldots, q$.
- These sums are ordered as follows: $S_{[1]}(v) \leq \ldots \leq S_{[q]}(v)$; paths corresponding to these sums are denoted by $\mathcal{P}_{[1]}(v), \ldots, P_{[q]}(v)$ respectively.
- Efficiency requires trade to take place with sellers in $\mathcal{P}_{[1]}(v)$ if $v_0 > S_{[1]}(v)$; if $v_0 \leq S_{[1]}(v)$ then trade does not occur.
- Note that this rule is not fully specified, but it will not matter.

Example

- Here $\mathcal{P}_{[1]}(v) = \{123\}$ and $S_{[1]}(v) = 19$. Efficiency requires trade with sellers 1, 2 and 3 if $v_0 > 19$.

Successful Mechanisms

Definition (First Best)

A mechanism achieves the *first-best* if it satisfies efficiency, IIR and BB.

Definition (Successful Mechanism)

A mechanism is *successful* if

(a) it is BIC with respect to some prior μ, and

(b) it achieves the first best.

Result: Sufficiency for Possibility when $q > 1$

Theorem

Let $\langle \Gamma, k, \mu \rangle$ be a land acquisition problem with $q > 1$. Suppose μ satisfies the following condition:

$$\underline{v}_0 \geq E\left(\sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) \right) - (k-1) E\left(S_{[1]}(v) \right).$$

Then there exists a successful mechanism with respect to μ.

Result: Necessity for Possibility when $q > 1$

Theorem

If there exists a successful mechanism with respect to μ, then

$$\underline{v}_0 > E\left(\sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) - (k-1)S_{[1]}(v) \middle| v \in \tilde{V}\right),$$

where

$$\tilde{V} = \{v \in [\underline{v}_0, \bar{v}_0] \times [\underline{v}, \bar{v}]^n : \underline{v}_0 > S_{[1]}(v) \text{ and } v_0 > S_{[1]}(\bar{v}, v_{-i}) \text{ for all } i \in \mathcal{P}_{[1]}(v)\}.$$

Result: Impossibility when $q = 1$

Theorem

Let $\langle \Gamma, k, \mu \rangle$ be a land acquisition problem with $q = 1$. There does not exist any successful mechanism.

Sketch of proof

- We use a result by Krishna and Perry (1998): there exists a successful mechanism if and only if the well-known VCG mechanism runs an expected budget surplus.
- The VCG mechanism is efficient.
- Let $SW(v) = v_0 - S_{[1]}(v)$ if $v_0 > S_{[1]}(v)$ and 0 otherwise.
- The buyer's VCG payment is given by $SW(\underline{v}_0, v_{-0}) - SW_{-0}(v_0, v_{-0})$.
- Seller i's VCG payment is given by $SW(\bar{v}, v_{-i}) - SW(v_i, v_{-i})$.

Proof contd...

- We explicitly calculate the VCG surplus of payments for all possible profiles.
- When $q = 1$, there is no strictly positive surplus at any profile.
- When $q > 1$, there is a subset of profiles where surplus may become strictly positive — the proof exploits this subset of profiles.

Further Explanations

- Since VCG is efficient, trade takes place with sellers in the feasible path with the lowest sum of valuations.
- A successful seller is *trade-pivotal* at a profile if trade takes place at his true valuation but not when he reports his highest valuation.
- The VCG receipt of the successful seller who is not trade-pivotal is higher than that of a successful seller who is trade-pivotal; unsuccessful sellers do not receive any payment.

Explanations..

- It follows that the sum of seller receipts cannot be more than in the case when all successful sellers are non-trade-pivotal.
- Further, the VCG payment of the buyer is at least as much as his minimum possible valuation.
- Consequently, the VCG mechanism must result in an expected surplus of payments if the buyer's minimum valuation is expected to be higher than the sum of VCG receipts of k non-trade-pivotal sellers.
- Conversely, if the VCG mechanism results in an expected surplus of payments, then the buyer at his lowest possible valuation is expected to be able to compensate sellers if all of them are non-trade-pivotal.

Illustration: Pivotal and non-Pivotal Sellers

- $n = 4$, $k = 3$ and $q = 2$; $\underline{v}_0 = 25$, $\bar{v}_0 = 35$, $\underline{v} = 0$, $\bar{v} = 10$. Let $v_0 = 26$.

Illustration...

- Sellers 1, 2 and 3 trade at v.
- If seller 1's valuation is 10 instead of 3, the sum of the valuations on paths $\{123\}$ and $\{234\}$ are 28 and 26 respectively. Hence trade does not take place at $(10, v_{-1})$, i.e., seller 1 is trade-pivotal at v.
- Sellers 2 and 3 are not trade-pivotal at v: if seller 2 has a valuation of 10, the sum of valuations on $\{123\}$ is 20 and trade can take place at $(10, v_{-2})$; same follows for seller 3.

Illustration: When all successful sellers are non-pivotal

- $v'_0 = 28$, , $v'_1 = 1$, $v'_2 = 2$, $v'_3 = 3$ and $v'_4 = 2$. Trade takes place at v' with sellers 1, 2 and 3. Note that trade also takes place when the buyer's valuation is the lowest possible, i.e., $\underline{v}_0 = 25$. Furthermore, no successful seller at v' is trade-pivotal.

Interpretation of the Main Result

- $\tilde{V}$ is the set of profiles v such that
  (i) trade takes place at $(\underline{v}_0, v_{-0})$ and therefore also at v, and
  (ii) all successful sellers are non-pivotal at v.
- Note that, v' belongs to $\tilde{V}$ but $v \notin \tilde{V}$.

Interpretation..

- Pick $v \in \tilde{V}$ and a successful seller i.
- If i's valuation changes to $\bar{v}$, trade still takes place and the sum of valuations of the successful sellers in the profile $(\bar{v}, v_{-i})$ is $S_{[1]}(\bar{v}, v_{-i})$.
- The sum of valuations of all successful sellers other than i at v is $S_{[1]}(v) - v_i$.
- The difference of these two terms, summed over all successful sellers, is $\sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(\bar{v}, v_{-i}) - (k-1)S_{[1]}(v)$.
- If the expectation of this term is at most $\underline{v}_0$, successful mechanisms exist. Conversely, if there exists a successful mechanism, then the expectation of this term, conditional on the profile belonging to $\tilde{V}$, is less than $\underline{v}_0$.

Numerical Example

- Line graph with $n = 4$, $k = 3$; seller valuations are iid $U[0, 10]$.
- Let $v_{[1]}^{14} = \min\{v_1, v_4\}$ and $v_{[2]}^{14} = \max\{v_1, v_4\}$. Then,

$$\begin{aligned} S_{[1]}(v) &= v_{[1]}^{14} + v_2 + v_3, \\ S_{[1]}(10, v_{[2]}^{14}, v_2, v_3) &= v_{[2]}^{14} + v_2 + v_3, \\ S_{[1]}(v_{[1]}^{14}, v_{[2]}^{14}, 10, v_3) &= v_{[1]}^{14} + 10 + v_3, \\ S_{[1]}(v_{[1]}^{14}, v_{[2]}^{14}, v_2, 10) &= v_{[1]}^{14} + 10 + v_2. \end{aligned}$$

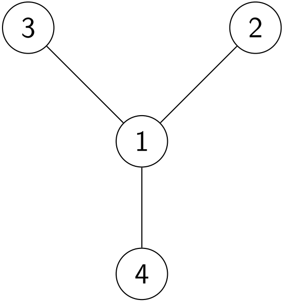
Therefore,

$$\sum_{i \in \mathcal{P}_{[1]}(v)} S_{[1]}(10, v_{-i}) - (k-1)S_{[1]}(v) = v_{[2]}^{14} + 20.$$

- Sufficiency condition is satisfied if $\underline{v}_0 \geq E(v_{[2]}^{14}) + 20 = 26\frac{2}{3}$.

Critical Sellers

- A seller i is *critical* if i belongs to every feasible path $\mathcal{P}_j$, e.g., sellers 2 and 3 in last example, or seller 1 in the figure below.



Rewriting the Conditions for Possibility

Theorem

Let $c(\Gamma)$ denote the set of critical sellers in $\langle \Gamma, k, \mu \rangle$; $q > 1$ implies $|c(\Gamma)| \leq k - 1$. There exists a successful mechanism if μ satisfies the following condition:

$$\underline{v}_0 \geq |c(\Gamma)|\bar{v} + E\left(T(v)\right)$$

where,
$T(v) = \sum_{i \in \mathcal{P}_{[1]}(v) \setminus c(\Gamma)} \left(S_{[1]}(\bar{v}, v_{-i}) + v_i\right) - \left(k - |c(\Gamma)|\right) S_{[1]}(v)$.

Necessary Condition

Theorem

If there exists a successful mechanism with respect to μ then

$$\underline{v}_0 > |c(\Gamma)|\bar{v} + E\left(T(v)\middle| v \in \tilde{V}\right)$$

where

$$\tilde{V} = \{v \in [\underline{v}_0, \bar{v}_0] \times [\underline{v}, \bar{v}]^n : \underline{v}_0 > S_{[1]}(v), v_0 > S_{[1]}(\bar{v}, v_{-i}) \forall i \in \mathcal{P}_{[1]}(v)\}$$

A Corollary

- Suppose there exists a successful mechanism with respect to μ. Then

$$\underline{v}_0 > |c(\Gamma)|\bar{v}.$$

- The count of critical nodes puts a lower bound on the support of the buyer's valuation essential for the existence of a successful mechanism.
- Since not all nodes on a feasible path can become critical, the existence of critical nodes does not preclude the attainability of the first best.

Without Contiguity

- If there is no contiguity requirement for the purchased plots, the buyer can purchase *any* set of k plots.
- This case is obtained when the underlying graph Γ is complete: there exists an edge between every pair of nodes.
- Efficiency: fix a valuation profile v. Let $v_{[i]}$ be i-th order statistic of the valuations of n sellers, so that $v_{[1]} \leq \ldots \leq v_{[n]}$. An efficient rule P^* is described as follows: for a profile v, trade occurs if $v_0 \geq \sum_{i=1}^{k} v_{[i]}$, and no-trade otherwise.

Possibility without Contiguity Requirement

A-I Suppose $n > k$. There exists a successful mechanism if μ satisfies the following condition:

$$\underline{v}_0 \geq kE\left(v_{[k+1]}\right).$$

A-II If there exists a successful mechanism then μ satisfies the following condition:

$$\underline{v}_0 > kE\left(v_{[k+1]} \mid (\underline{v}_0 > \sum_{j=1}^{k} v_{[j]}) \cap (v_0 > \sum_{j=2}^{k+1} v_{[j]})\right).$$

B. Suppose $n = k$. Then there does not exist a successful mechanism.

Example

- Seller valuations are distributed uniformly in $[0, 1]$.
- Assumption NT requires $\bar{v}_0 > 0$ and $\underline{v}_0 < k$.
- In this case, $E\left(v_{[k+1]}\right) = \frac{k+1}{n+1}$.
- According to our result, $\underline{v}_0 \geq \frac{k(k+1)}{n+1}$ guarantees the existence of successful mechanisms
- For instance, if $n = 2$ and $k = 1$, $\underline{v}_0 \geq \frac{2}{3}$ is the required condition.
- Convergence: Since $\frac{k(k+1)}{n+1} \to 0$ as $n \to \infty$, it becomes easier to satisfy the sufficient condition as the number of sellers increase.

Takeaway

- It *is* possible to construct successful mechanisms for a large class of Land Acquisition problems fulfilling certain criteria.
- Elsewhere we have shown that simple conditions on supports of valuations ensure that weak budget balance can be achieved by VCG in the limit as number of sellers become large.

THANK YOU!